A dual-weighted trust-region adaptive POD 4D-VAR applied to a FEM shallow-water equations model

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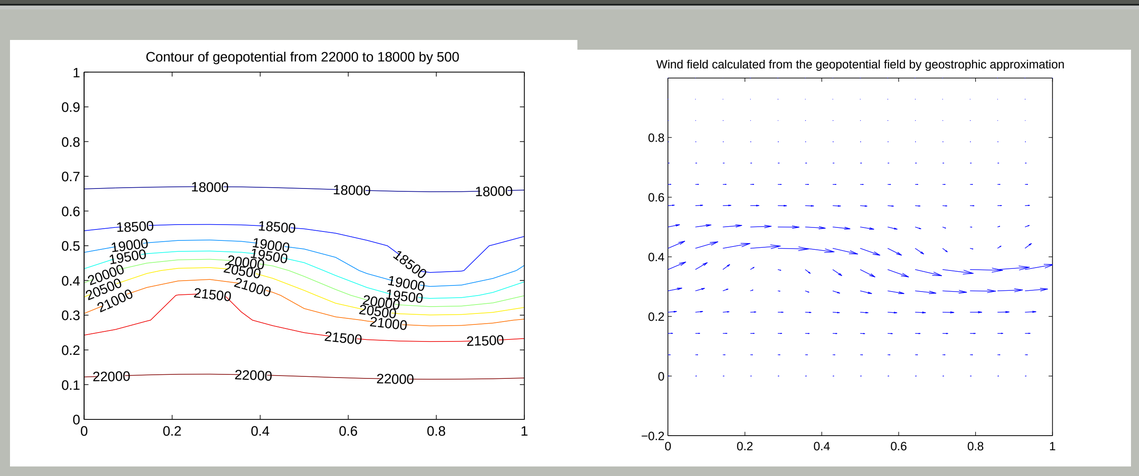
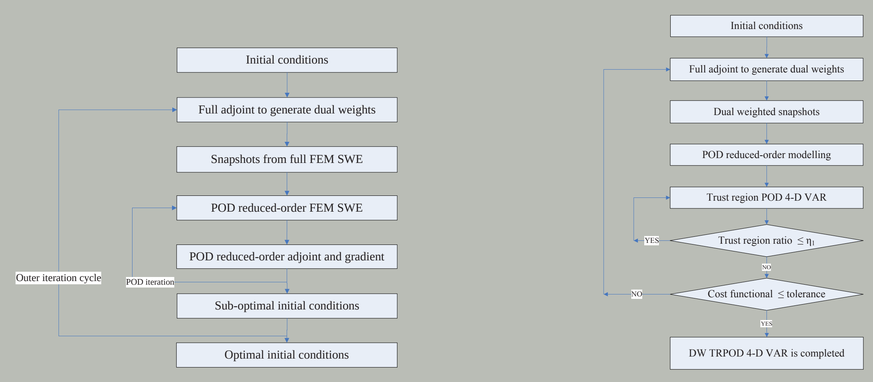
Abstract

A limited-area finite-element model of the nonlinear shallow-water equations is used to solve an inverse problem where the initial conditions are optimized in presence of observations being assimilated in a time interval. We then consider a reduced-order model of the above inverse problem, based on proper orthogonal decomposition (POD), referred to as POD 4-D VAR. A dual-weighted method for efficient POD snapshot selection is coupled with a trust-region adaptivity approach. Numerical results obtained point to an improved accuracy in all metrics tested when dual-weighing choice of snapshots is combined with POD adaptivity of the trust-region type. Results of ad-hoc adaptivity of the POD 4-D VAR turn out to yield less accurate results than trust-region POD when compared with the high-fidelity model. Directions of future research will be outlined.

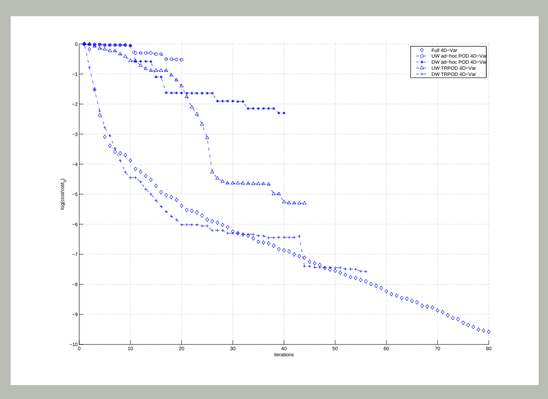
Grammeltvedt I initial condition of SWE model in a limited-area

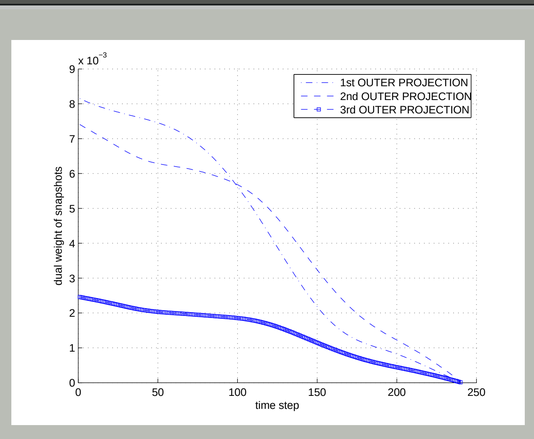


Flowchart of the methodology combining dual weighed snapshots and TRPOD adaptivity



Comparison of the performance of minimization of cost functional and dual weights of snapshots data determined by the full adjoint

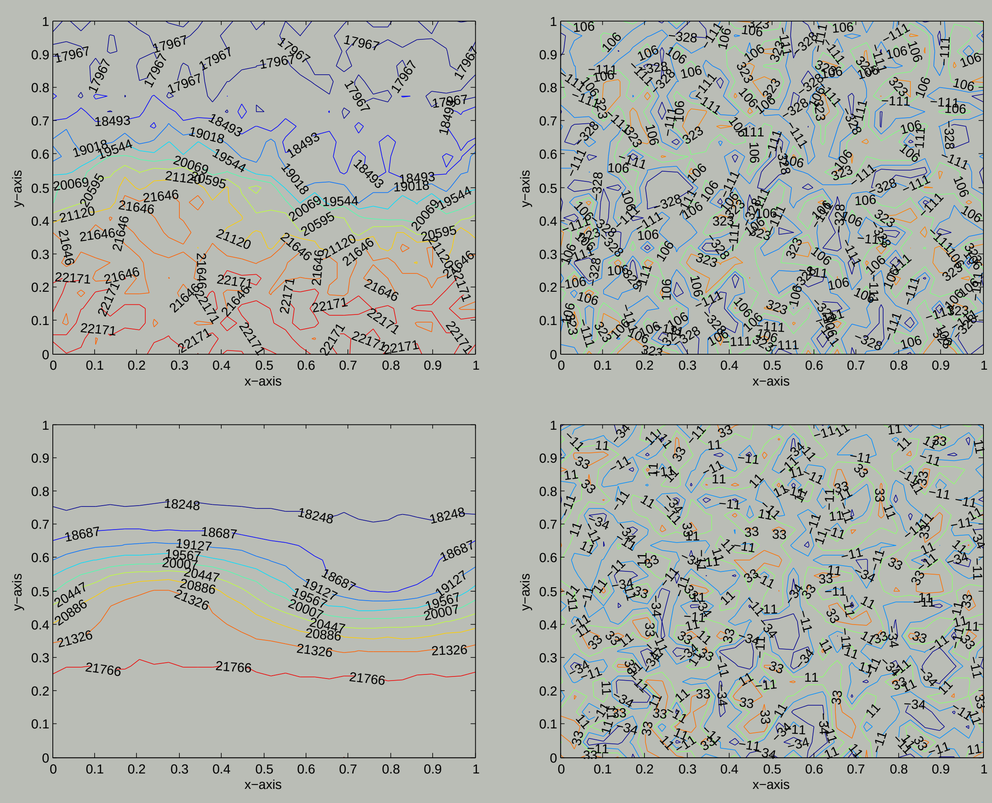




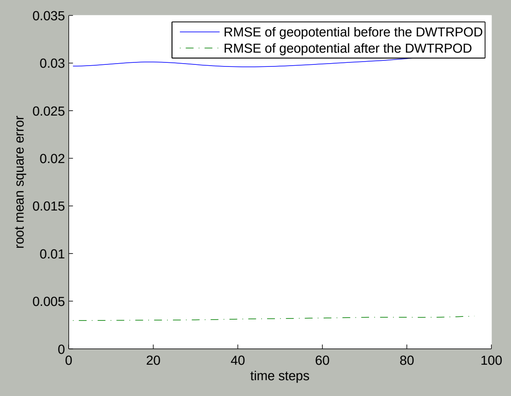
POD 4-D Var	ADPOD	DWAHPOD	TRPOD	DWTRPOD	Full
Iterations	**22**	**42**	**46**	**57**	80
Outer projections	**2**	**6**	**10**	**12**	N/A
Error	$\mathbf{10^{-1}}$	$\mathbf{10^{-2}}$	$\mathbf{10^{-5}}$	$\mathbf{10^{-8}}$	$\mathbf{10^{-10}}$
CPU time (s)	**15.2**	**38.7**	**121.2**	**142.8**	**222.6**

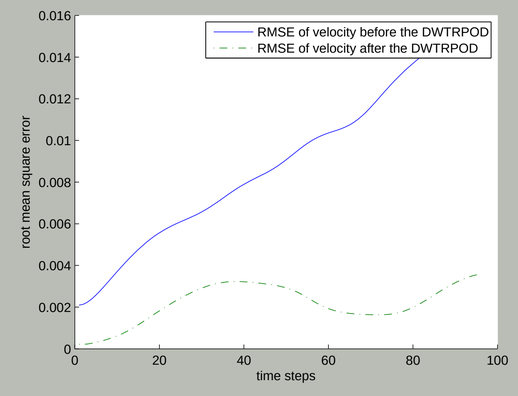
Table: Comparison of iterations, outer projections, error and CPU time for ad-hoc POD 4-D Var, ad-hoc dual weighed POD 4-D Var, trust-region POD 4-D Var, trust-region dual weighed POD 4-D Var and the full model 4-D Var.

Errors between the retrieved initial geopotential and true initial geopotential applying dual weighted trust-region POD 4-D Var

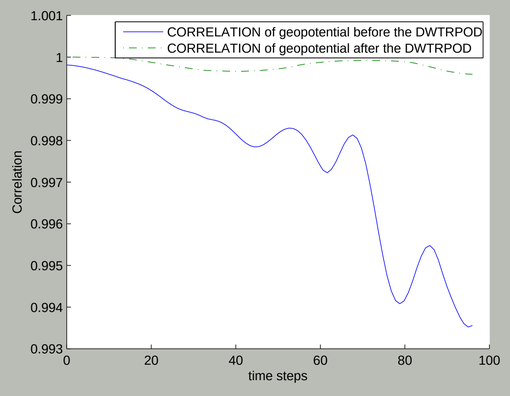


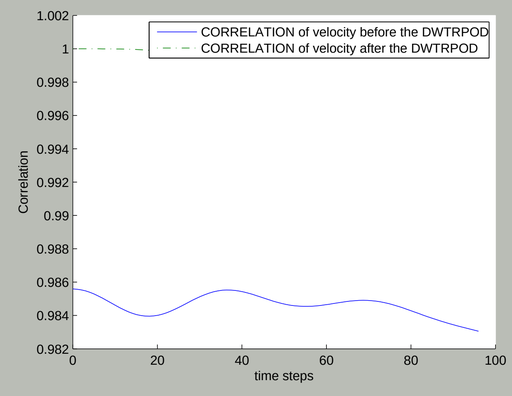
RMSE after dual-weighted trust-region POD 4-D VAR



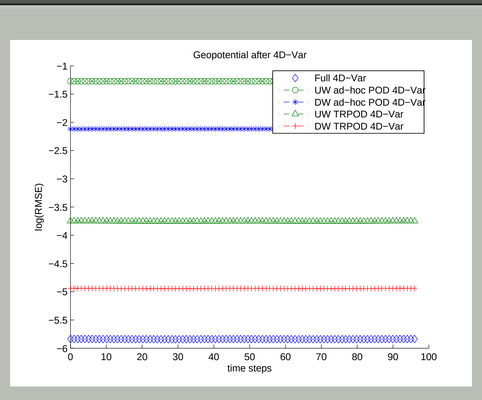


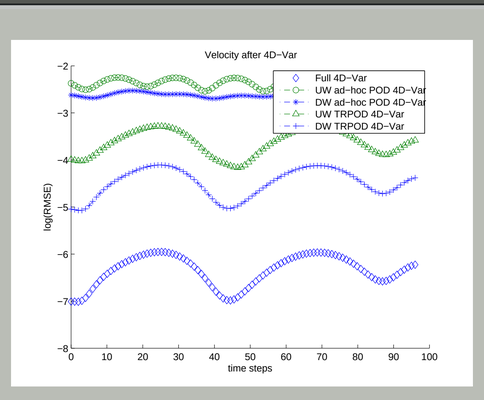
Correlation after dual-weighted trust-region POD 4-D VAR





Comparison of RMSE between full model and POD model among different types of 4D-VAR





Comparison of correlation between full model and POD model among different types of 4D-VAR